Users Guide for the Cassini Radio Science ionospheric electron density profiles data set for both Saturn and Titan

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This users guide is intended as an aid to understanding the ionospheric electron density profiles determined by radio occultations of the Cassini spacecraft by Saturn and Titan during the course of Cassini's 13 year mission from 2004-2017. I will not go into any detail about the techniques used to collect Radio Science data from the Cassini spacecraft; that can be found in the *Cassini Radio Science User's Guide* by S. Asmar *et al.* available elsewhere. I will write specifically about this data set.

This data set is split in two by an unfortunate event. December 2014 marked the passing of Arvydas Kliore, Cassini Radio Science team leader and the person who computed ionospheric profiles from the collected radio science data from occultations. Kliore finished all of the Titan ionospheric profiles up to T102 (targeted Titan encounter 102, on orbit 205) which occurred on 18 June 2014, and all of the Saturn occultations up to the orbit 191 occultation which occurred on 31 May 2013. Kliore was working on but had not completed the orbit 197 occultation of 1 September 2013 when he passed away.

Subsequently I was given a collection of files, mysterious and undocumented, and asked to try to understand and convert them into a format suitable to be archived in the PDS. I was also asked to find ionospheric electron density profiles for the remainder of the Saturn and Titan occultations. (My main focus is to compute the structure of the neutral atmospheres of Titan and Saturn well below the ionosphere.) My first task was to understand how the electron density profiles are determined. I knew that the technique used, the "dual frequency" technique, used any two of the three frequencies that Cassini simultaneously transmitted and a ground station received, but it is suprisingly hard to find any details about the technique published anywhere. There's brief mention of it in [4] with, as I determined, an error in the leading constant, but nowhere could I find any details on the technique. However, it was straightforward to derive the technique, which I will present below along with two other techniques, one approximate and one which we use to remove ions from neutral atmosphere profiles which can be simply rearranged to find the electron density profile by removing the neutral atmosphere. I used the latter two techniques as a sanity check on my work and to make sure the assumption of spherical symmetry (discussed below) was not causing errors when used on Saturn's oblate atmosphere.

So, although I reproduced every Titan and Saturn electron density profiles using my own techniques, everything in this data set up to and including T102 and the orbit 191 Saturn occultation is simply Arv Kliore's electron density profiles repackaged into a PDS friendly format. (Kliore was the expert. I'm a beginner.) Everything after (T117 on orbit 232, T119 on orbit 235, and orbit 197 and after for Saturn) are my determination of the electron density profiles. The differences between the earlier and later profiles (vertical resolution, etc.) are entirely due to that. I cannot be certain that Kliore used exactly the dual frequency technique I present below, although I know that he used a dual frequency technique. When I determine a profile (electron density or neutral atmosphere) I tend to pick a single receiving station and concentrate on that, while Kliore seemed to somehow amalgamate the results from every pair of frequencies at, if more than one, every receiving station. There is still much in his files that I don't understand. One, which will be discussed in more detail below, is exactly what the altitudes mean in his Saturn electron density profiles. When I redetermined the Saturn profiles, I got profiles that were nearly identical to Kliore's in structure but "shifted" in altitude. This "Saturn altitude offset" problem will be discussed in more detail below.

There's another split in this data set caused by a hardware event aboard the spacecraft. In December 2011, the Ultra Stable Oscillator (USO) aboard Cassini failed. The USO was used as the frequency reference for all occultations. The USO generated a very stable monotone signal in the X band, and that signal was coherently multiplied by electronics aboard the spacecraft to produce an S band signal (multiply by 3/11) and a Ka band signal (multiply by 3.8). All three frequencies were transmitted simultaneously. After the USO failed, all subsequent occultations were done in two-way mode, where a very stable oscillator on the ground was used to send a signal (in the X band at about 7.2 GHz) to the spacecraft. The signal received by the spacecraft was immediately multplied by 880/749 and that signal was used as a replacement for the USO. We had control of the uplink frequency and using a model atmosphere computed the uplink frequency so that the frequency received by the spacecraft was as close as constant as possible in its rest frame, but it wasn't possible to do this exactly. This complicates the analysis of the neutral atmosphere a great deal, but fortunately the differential frequency technique is ideally suited to avoid all of the problems caused by referencing to the time varying uplink frequency rather than the USO, and the analysis goes exactly as before. (The other two techniques discussed below, however, cannot be used for two-way occultations.)

1 Differential Frequency Technique

The differential frequency technique uses two frequencies simultaneously to determine the structure of an ionosphere. This is done to remove non-dispersive effects, such as glitches in the USO in one-way mode or the variation in time of the uplink frequency as seen by the spacecraft in two-way mode, and isolates the effect of free electrons, which is strongly frequency dependent, on rays traversing the ionosphere. This assumes that you have a time series of the frequencies received by a single receiving antenna at two different frequencies. As noted above, Cassini generally transmitted at three different frequencies during occultations, S (2.3 GHz), X (8.4 GHz) and Ka (31.9 GHz).

1.1 Fundamentals

We begin with the equation relating the frequency received on the earth to the frequency transmitted by the spacecraft

$$\frac{f_a}{f_s} = \left(\frac{1 - (\hat{\mathbf{n}}_a \cdot \mathbf{v}_a/c)}{1 - (\hat{\mathbf{n}}_s \cdot \mathbf{v}_s/c)} \right) \tag{1}$$

Here f_a is the frequency received by the DSN antenna, f_s is the frequency transmitted by the spacecraft (either referenced to the USO or to an uplink signal), $\hat{\mathbf{n}}_s$ and $\hat{\mathbf{n}}_a$ are unit vectors tangent to the ray transmitted by the spacecraft and that ray received by the DSN, respectively, $\mathbf{v}_s$ is the velocity of the spacecraft and $\mathbf{v}_a$ is the velocity of the DSN antenna. For clarity here, we work only to first order in v/c and leave out second order relativistic effects, which we retain in the actual analysis. This equation is valid in both the X and S band, and at the spacecraft $f_S/f_X = 3/11$. (X and S bands are used for illustration; any two bands will do so long as the ratios between the frequencies at the spacecraft are constant.)

Now we look at the difference

$$\Delta f \equiv f_{aS} - \frac{3}{11} f_{aX}, \tag{2}$$

where f_{aS} and f_{aX} are the frequencies received by the DSN in S band and X band respectively. If the path of the ray is entirely in vacuum, then this will be zero. In addition, if the index of refraction of the atmosphere between the spacecraft and the DSN is non-dispersive, so that it's the same for X and S band, then this will also be zero, because the rays will follow exactly the same path. Using equation 1 in equation 2, and remembering that if refraction is dispersive the initial ray direction in the X band need not be the same as the initial ray direction in the S band, this becomes

$$\Delta f = f_{sS}(1 - (\hat{\mathbf{n}}_a \cdot \mathbf{v}_a/c)) \left[\frac{1}{1 - (\hat{\mathbf{n}}_{sS} \cdot \mathbf{v}_s/c)} - \frac{1}{1 - (\hat{\mathbf{n}}_{sX} \cdot \mathbf{v}_s/c)} \right] \tag{3}$$

or

$$\Delta f = f_{aS} \frac{\Delta \mathbf{n} \cdot \mathbf{v}_s/c}{1 - \mathbf{n}_{sS} \cdot \mathbf{v}_s/c} \tag{4}$$

and we have defined $\Delta \mathbf{n} \equiv \mathbf{n}_{sS} - \mathbf{n}_{sX}$. Here we have used the fact that Saturn/Titan are much farther from the Earth than from Cassini, so there will be no difference in the arrival unit vector $\mathbf{n}_a$ between X and S bands even though the ionosphere is dispersive. So we can find the difference between the initial X and S band ray directions from

$$\begin{aligned} v_s(\cos\alpha_s - \cos\alpha_x) &= v_s(2\sin(\tfrac{1}{2}(\alpha_s+\alpha_x))\sin(\tfrac{1}{2}(\alpha_s-\alpha_x))) \\ &\approx v_s \sin\alpha_s(\alpha_s - \alpha_x) \end{aligned} \tag{5}$$

where α is the angle between the projection of the spacecraft's velocity in the plane of propagation v_s and the ray direction. (Here we assume that both the X and S band rays propagate in the same plane in spite of the dispersion due to the ions, which will be true if the ionosphere is spherically symmetric).

Next we look at the geometric optics equation for the path of a ray

$$\frac{d}{ds}\left(\mathrm{n}\frac{d\mathbf{r}}{ds}\right) = \nabla \mathrm{n} \tag{6}$$

where n is the index of refraction, $\mathbf{r}$ is the position vector, and s is a length of the ray from an initial point. Now consider the change in the vector $\mathbf{r} \times \mathrm{n}\mathbf{s}$, where $\mathbf{s} = d\mathbf{r}/ds$ is tangent to the ray at any point along the ray. This is

$$\frac{d(\mathbf{r}\times \mathrm{n}\mathbf{s})}{ds} = \mathbf{r} \times \frac{d(\mathrm{n}\mathbf{s})}{ds} + \frac{d\mathbf{r}}{ds} \times \mathrm{n}\mathbf{s} \tag{7}$$

Since $d\mathbf{r}/ds = \mathbf{s}$ the second term vanishes. Now, assume that n is spherically symmetric and so depends only on the radial coordinate r. Then the first term, using equation 6 becomes

$$\mathbf{r} \times \nabla \mathrm{n} \tag{8}$$

which is zero because by the assumption of spherical symmtery ∇n is parallel to $\mathbf{r}$. So $\mathbf{r} \times \mathrm{n}\mathbf{s} = $ constant (Bouguer's law), or $r\mathrm{n}\sin\phi = a$, where a, the ray asymptote, is a constant, and ϕ is the angle between the ray tangent $\mathbf{s}$ and $\mathbf{r}$.

Now we will proceed to obtain the relationship between the bending angle β and the refractivity n. Taking Bouguer's law and differentiating by r, we find that

$$\frac{d\phi}{dr} = -\tan\phi\left(\frac{d\ln \mathrm{n}}{dr} - \frac{1}{r}\right) \tag{9}$$

Using Bouguer's law $\sin\phi = a/(nr)$, so

$$\tan\phi = \frac{a}{\sqrt{(nr)^2 - a^2}} \tag{10}$$

Now the second term in equation 9 is simply the result for a straight line path, as would obtain when the index of refraction n is 1. We're looking for the deviation of the ray from the straight line path, more particularly, the difference between the initial ray direction and the final ray direction outside the atmosphere. But this is just, because of the symmetry of the ray path around the periapsis

$$\beta = -2\int_{r_p}^{\infty}(\frac{d\phi}{dr} - (\frac{d\phi}{dr})_{\mathrm{line}}) = -2a\int_{r_p}^{\infty}\frac{d\ln\mathrm{n}}{dr}\frac{dr}{\sqrt{(\mathrm{n}r)^2 - a^2}} \tag{11}$$

where $r_p = a/\mathrm{n}$ is the ray periapsis.

1.2 The Abel transform

Now we look at the difference between the bending angle for the S band ray and the bending angle for the X band ray, $\beta_S - \beta_X$. We note that $\mathrm{n} = 1 - N_e A/f^2$ in the ionosphere, where N_e is the electron density in cm^{-3}, f is the frequency in Hz, and the constant $A = 4.03 \times 10^7$. So the difference in the bending angle is

$$\begin{aligned}\beta_S - \beta_X = \quad & 2a_S \left[\int_{r_{pS}}^{\infty} \frac{A}{\mathrm{n}_S f_S^2} \frac{dN_e}{dr} \frac{dr}{\sqrt{(\mathrm{n}_S r)^2 - a_S^2}} \right] \\ & -2a_X \left[\int_{r_{pX}}^{\infty} \frac{A}{\mathrm{n}_X f_X^2} \frac{dN_e}{dr} \frac{dr}{\sqrt{(\mathrm{n}_X r)^2 - a_X^2}} \right]\end{aligned} \tag{12}$$

In the second integral, define a new variable $R = (\mathrm{n}_X/\mathrm{n}_S)r$. Then it becomes

$$\int_{R_{pX}}^{\infty} \frac{A}{\mathrm{n}_X f_X^2} \frac{dN_e}{dR} \frac{dR}{\sqrt{(\mathrm{n}_S R)^2 - a_X^2}} \tag{13}$$

But $R_{PX} = \mathrm{n}_X r_{pX}/\mathrm{n}_S = a_X/\mathrm{n}_S = r_{pS}$, where we will from here on ignore the slight difference between a_X and a_S (and therefore r_{pS} and r_{pX}) which occurs because the medium is dispersive. So we can replace the integration variable R by r in the second integral, and we get

$$\beta_S - \beta_X = 2a \int_{r_{jS}}^{\infty} \left(1 - \frac{\mathrm{n}_S}{\mathrm{n}_X} \frac{f_S^2}{f_X^2}\right) \frac{A}{\mathrm{n}_S f_S^2} \frac{dN_e}{dr} \frac{dr}{\sqrt{(\mathrm{n}_S r)^2 - a^2}} \tag{14}$$

Now we will assume that $\mathrm{n}_S/\mathrm{n}_X = 1$ and $n_S = 1$ to the accuracy we need. (The refractivity of the ionosphere is on the order of 10^{-2} even in the ionospheric layers of Saturn where the electron density is highest, so the contribution to the index of refraction is about -10^{-8}.) So to high accuracy

$$\beta_S - \beta_X = 2a \left(1 - \frac{f_S^2}{f_X^2}\right) \int_{r_p}^{\infty} \frac{A}{f_S^2} \frac{dN_e}{dr} \frac{dr}{\sqrt{r^2 - a^2}} \tag{15}$$

The solution to this is found by an Abel transform:

$$N_e = -\frac{1}{A\pi} \frac{f_S^2}{1 - (f_S/f_X)^2} \int_{a_j}^{\infty} \frac{(\beta_S - \beta_X)da}{\sqrt{a^2 - a_j^2}} \tag{16}$$

where $a_j = \mathrm{n}(r_p)r_p$ Since

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) \tag{17}$$

this integral may be done by parts to get the form that we use

$$N_e = -\frac{1}{A\pi} \frac{f_S^2}{1 - (f_S/f_X)^2} \int_{\beta_S - \beta_X}^{0} \ln\left(\frac{a}{a_j} + \sqrt{\left(\frac{a}{a_j}\right)^2 - 1}\right) d\beta \tag{18}$$

Because the arrival vector at the receiving antenna is the same for both, $\beta_S - \beta_X = \alpha_s - \alpha_x$ in equation 5.

Now in the above several times I have made the assumption of spherical symmetry, the first being when I assumed the ray path stayed in a plane, and later on that the index of refraction n is a function of radius only. In a spherically symmetric atmosphere both of these are true. Rays from the spacecraft to the earth stay in a plane defined by the center of the target (center of symmetry), the spacecraft's position and the position of the receiving antenna. (Of course, assuming $n = n(r)$ where r is the distance from the center of the target is the definition of assuming spherical symmetry.)

But it has been known since the early days of solar system exploration that using an Abel transform technique assuming spherical symmetry on an oblate atmosphere such as Jupiter or Saturn leads to wildly incorrect results ([2]) for the structure of the neutral atmosphere without modification. (If one insists on using an Abel transform the typical modification is to use a so called "radius of curvature" and do the analysis using a center that is not the center of the target. Instead, for our neutral atmospheres work we use a ray tracing technique ([3]) that can handle both oblateness and differential rotation when solving for the neutral atmosphere.) So why can we assume spherical symmetry here? The index of refraction of the ionosphere is very small, n deviates from 1 by about 10^{-8} in the part of the ionosphere where the electron density is greatest, so the ray's barely bend. They are still essentially straight lines, and the Abel transform actually still gives accurate results for the electron density.

2 Other techniques to determine electron density

When solving for what I'm most interested in, the structure of the neutral atmosphere, I obtain a profile of refractivity ($N = 10^6(n-1)$) vs. a vertical coordinate (altitude above the surface for Titan, the gravitocentrifugal potential for Saturn). For Titan we assume spherical symmetry and use an Abel transform techinque to find the refractivity from the surface of Titan to an altitude to where the refractivity is essentially zero (index of refraction $n = 1$). For Saturn as I noted above assuming spherical symmetry leads to wildly incorrect results, so for Saturn we use a a ray tracing technique ([3]) which can handle both oblateness and differential rotation due to both the bulk rotation of Saturn and differential winds in the atmosphere.

For technical reasons these profiles extend well above the point at which we usually start our hydrostatic equilibrium integration to get pressure, temperature, etc. One simple way to get the electron density is to assume that in a vertical region the entire refractivity is due to electron density

$$N_e = -f^2 N / 4.03 \times 10^{13} \text{cm}^{-3}. \tag{19}$$

where f is the frequency in Hz. This actually works quite well if you can identify the ionosphere, because in the ionosphere it actually is the case that essentially all of the refractivity is due to free electrons. It's usually possible to locate the ionosphere simply by looking where the refractivity N is negative. This is how we determined the structure of the ionospheres of Jupiter ([1]) and its satellites during the Galileo mission where only one frequency was sent and, due to the failure of the high gain antenna, the ionosphere was all we were able to do. (And, as is the case for the dual frequency technique, assuming spherical symmetry and using an Abel transform worked well for just the ionosphere.)

If one has refractivity profiles independently found from two different frequencies recorded at the same antenna, there's a similar technique that can be used that doesn't require you to find the ionosphere by eye. Suppose that we have two refractivity profiles, N_S from S-band data and N_X from X band data recorded at the same antenna. (For Cassini, X and S were usually recorded simultaneously using a 70 m antenna, and X and Ka were simultaneously recorded at a 34 m antenna.) Now we assume that

$$N = N_0 + N_i = N_0 - 4.03 \times 10^{13} N_e / f^2 \tag{20}$$

where N_0 is the non-dispersive part of the refractivity that's frequency independent due to neutral molecules, and N_i is the frequency dependent part of the refractivity due to electrons. So $N_S = N_0 + N_{iS}$, and $N_X = N_0 + N_{iX}$. For the neutral atmosphere we solve these two equations for N_0 and eliminate the effects of electrons, but we can just as easily eliminate N_0 and solve for the electron density. A little algebra and we find

$$N_e = -\frac{f_S^2 (N_S - N_X)}{4.03 \times 10^{13} (1 - (3/11)^2)} \mathrm{cm}^{-3} \tag{21}$$

where I've used that fact that for Cassini $f_X = 11/3 f_S$.

3 The Saturn altitude offset problem

Arv Kliore, in his papers and in the files I have, plots electron density as a function of altitude. No one seems to know what that altitude scale is referenced to (what is 0 km?), and he doesn't seem to say anywhere.

On Titan it's easy. Everyone uses a sphere of 2575 km. Now Titan has terrain and is not a strictly a sphere but it's very close to being spherical. When someone says "maximum electron density is at 600 km" everyone knows that means that it's 3175 km away from the center of Titan.

Saturn doesn't have a solid surface, so there's nothing equivalent. Most people use the "1 bar" surface (or maybe some other fixed pressure surface). The problem is you have to be able to solve for the atmospheric structure to find the actual 1 bar surface. NAIF (Navigation Ancillary Information Facility) has something called the "NAIF reference ellipsoid" but it's not the actual 1 bar surface, which is not an ellipsoid. The NAIF surface can be as much as 200 km away from the actual 1 bar surface; it's only guaranteed to be correct at the pole and the equator. When I started recomputing the electron density profiles for

Saturn, I found that I got profiles that looked very similar to Kliore's but were often (not always) offset in altitude (and at first I referenced to the actual 1 bar surface that I found from the neutral atmosphere profile I produced from the same radio occultation data). Figure 1 shows an example of one of the largest offsets I found, the rev 44 ingress occultation.

So I decided I had to do something about the problem of not knowing what Kliore's altitudes meant before archiving the profiles. What I eventually decided to do was reference to the 1 bar NAIF ellipsoid (an ellipsoid with a 60268 km semi-major axis and 54364 km semi-minor axis), since everyone can compute that easily, even though it's not an actual surface of constant pressure and deviates from the actual 1 bar surface. Then the user of these profiles can know exactly where in space the electron density peaks are. I knew exactly where the peaks I computed were in space, since I knew what coordinate system I was using, so I offset Kliore's altitudes until my highest major peak (first peak encountered while descending from free space) aligned with his. (That peak was the target, but the other peaks usually also aligned after the shift). The altitude is now referenced to the NAIF reference ellipsoid, so 3000 km in a .TAB file means "3000 km above the NAIF reference ellipsoid at that latitude". (I use the usual radio science convention that the "latitude" of a layer is the latitude of the periapsis of ray which first encounters the layer. Ionospheric layers extend in both latitude and longitude, and subsequent rays that extend deeper into the ionosphere will also cross the layer in question. Ionospheric layers on Saturn will be shaped not only by the oblate atmosphere lying below but also by Saturn's magnetic field.) Rather than just shifting the altitude, putting that altitude in the .TAB files and not saying anything about it, I also put the offset I used in the .LBL files, so if someone wanted to reconstruct Kliore's original altitude scale they could do it by just simply subtracting the offset from the altitudes in the .TAB file. Now you know exactly where in space the electron density peaks are, but if you need Kliore's original altitude scale for some reason, you can reconstruct it.

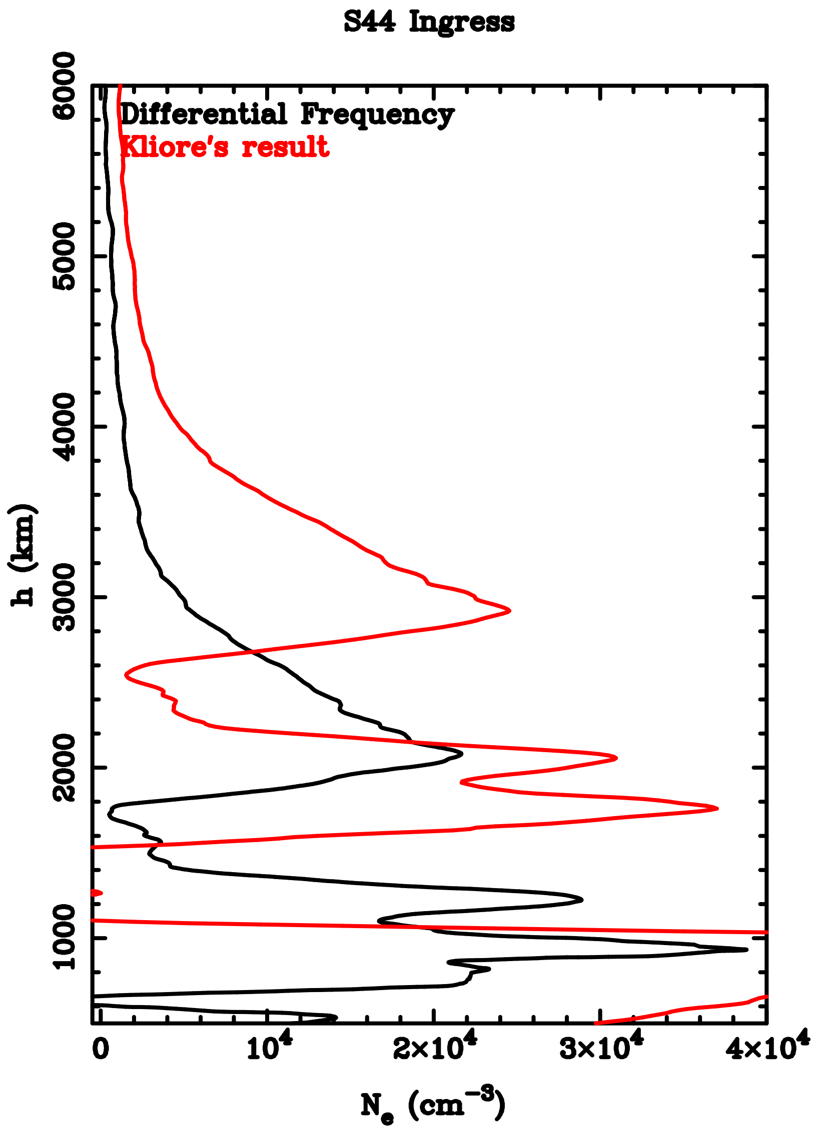

Figure 1: Electron density vs. altitude for the orbit 44 ingress occultation. My "differential frequency" result is referenced in altitude to the 1 bar NAIF reference ellipsoid. Kliore's profile uses his original altitudes, showing the problem I had to solve.

References

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